

Arizona Department of Education John Huppenthal, Superintendent of Public Instruction

# Arizona Adult Education Mathematics Content Standards

Revised 2012

www.azed.gov/adult-ed

## **Mathematics Content Standards Introduction**

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on that promise, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." The Arizona Adult Education Standards, building on the Common Core State Standards for Mathematics, are a substantial answer to that challenge.

It is important to recognize the "fewer standards" are no substitute for focused standards. Achieving "fewer standards" would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are: articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g. the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties). These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the "sequence of topics and performance" that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing "sequenced obstacles and challenges for students…absent the insights about meaning that derive from careful studying of learning, would be unfortunate and unwise." In recognition of this, the development of these Standards began with the Common Core State Standards—research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time. They provide framework those points out the sequence of topics through the levels.

## These topics include:

**Number Sense:** (Properties of numbers; computation and reasoning; rounding, estimation, and reasonableness of results; fractions, ratios, and proportions; exponents and square roots)

Data Collection and Responsibility: Data collection and analysis, measures of central tendency, properties of the normal curve; and probability)

**Algebra:** (Patterns, solving simple equations, representing linear functions, and properties of functions; solving linear equations and inequalities; solving systems of equations; solving quadratic equations; calculating with polynomial expressions, interpreting equations and inequalities)

**Geometry:** (Characteristics, constructions, and properties of figures; angles; coordinate geometry; properties of similarity and congruence; concept of  $\pi$  and circular figures; using properties and theorems to make conjectures and solve problems; right triangles and trigonometric functions)

Measurement: (Expressing, comparing, and converting measurements using appropriate units and tools)

# **Understanding Mathematics**

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a+b)(x+y) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands mathematics, and may have a better chance to succeed as a less familiar task such as expanding (a+b+c)(x+y). Mathematical understanding and procedural skills are equally important, and both are assessable using mathematical tasks of sufficient richness.

All students must have to opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the onset so all students can meet the goal of college and career readiness for all students.

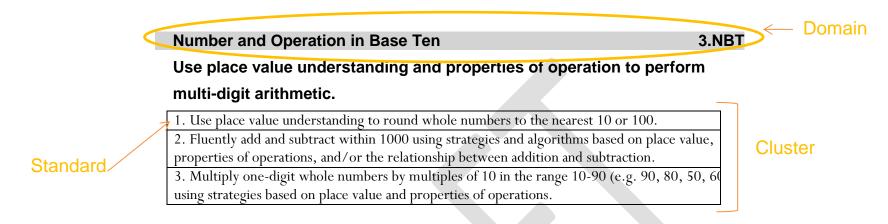
# How to Read the CCSS Grade Level Coding Schema

The Arizona Adult Education standards have the CCSS grade level codes attached so that each can be traced back to its original position in the Common Core State Standards.

Standards define what student should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.



The order of the standards does not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards of a given level, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What adult students can learn at any particular level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know...should next come to learn..." However presently this approach is unrealistic — not least because existing education research cannot specify all such learning pathways. Of necessity, therefore, level placements for specific topics have been made on the basis of state and international comparison sand the collective experience and collective professional judgment of educators, researchers and mathematicians from right here in Arizona.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time to recognize that standards are not just promises to our students, but promises we intend to keep.

# Mathematics I Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly,

accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief of diligence and one's own efficacy).

#### 1. Make sense of problems and persevere in solving them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry point to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. The monitor and evaluate their progress and change course if necessary. More advanced students might, depending on the context of the problem, transfer algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Beginning students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2. Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* — to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents — and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, nit just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3. Construct viable arguments and critique the reasoning of others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the argument of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Beginning students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until

later in a student's understanding. Later, students learn to determine domains to which an argument applies. Students at all levels can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### 4. Model with mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In the beginning stages, this might be as simple as writing an addition equation to describe a situation. A mid-level student might apply proportional reasoning to plan a school event or analyze a problem in the community. A more advanced student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### 5. Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students are various levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6. Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In beginning levels students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

#### 7. Look for and make use of structure

Mathematically proficient students look closely to discern a pattern or structure. Beginning students, for example, might notice that three and seven more is the same amount as a seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see [7x8] equals the well-remembered [7x5+7x3], in preparation for learning about the distributive property. In the expression  $x^2+9x+14$ , older students can see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5-3(x-y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

#### 8. Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Mid-level students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, more advanced students night abstract the equation (y-2)/(x-1)=3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1),  $(x-1)(x^2+x+1)$ , and  $(x-1)(x^3+x^2+x+1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

# Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout their education experience. Designers of curricula, assessments, and professional development should all attend to the need to connect mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of a procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These point of intersection are intended to be weighted toward central and

generative concepts in the mathematics curriculum that most merit time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and adult student achievement on mathematics.

# AZ Mathematics Standards - ABE through ASE - Before and After

This report details the changes that will take place in the AZ Adult Basic Education standards with adoption of the new ABE/ASE standards based on the Common Core State Standards. It describes the differences: what is new to each level and what has been removed from the existing standards. In the lists below the new college- and career-ready ABE standards based on the Common Core State Standards were compared with the existing ABE-ASE standards.

## Major Shifts in Beginning ABE:

When comparing the new Beginning ABE standards to the existing, there is an overall slight reduction in the number of concepts at this level, with very few new concepts introduced. However some of the existing concepts are subsumed by a new, enlarged version of standard. (e.g., "Recognizes and uses the following mathematical symbols: +, -, =. " is replaced with "Selects the appropriate operation (+, -, =) to solve specific problems involving whole numbers.")

The biggest overall shift for the new Beginning ABE is that there is no Algebra section per se. All have been either moved to Beginning ABE – Number (e.g. skip counting and writing number sentences) or to ABE I (e.g. recognizing/extending patterns and classifying objects by attribute). The move to Number at the same level changes the conceptual application of the topics and focuses students' study. Detailed descriptions of how the new standards are changed when compared to the existing can be found below:

## Beginning ABE - Number Sense and Computation

- Count forward from any number.
- Skip count by 2s, 5s, and 10s. (This was moved to Number from Algebra.)
- Selects appropriate operation to solve a specific problem.
- Make change using different coins and currency.
- Writes and solves number sentences. (Moved from Algebra.)
- Using the properties of operations (commutative, associative) for computation, estimation, and checking results. (Moved here from ABE I.)

## Removed from this level and/or domain:

- Recognize when estimate is reasonable.
- Defining addition and subtraction through demonstration.
- Recognize and use correct symbols for number sentences (+, -, =).

## **Beginning ABE – Data Collection and Probability**

#### *New to this level and/or domain:*

• Classify and record data from a sample set. (Sample set was not used in the old standards. The idea of 'record' was described as 'display.')

## Removed from this level and/or domain:

- "Construct a display" was dropped in favor of record data.
- Describe the relationship displayed in simple graphs, charts, etc.

## Beginning ABE - Algebra

#### *New to this level and/or domain:*

There are no concepts added to the new Beginning ABE – Algebra.

## Removed from this level and/or domain:

- All Algebra standards were moved or deleted from Beginning ABE including:
  - O Skip counting moved to Number.
  - $\circ\quad$  Writes number sentences moved to Number and also to ABE I.
  - O Recognize and extend patterns moved to ABE I.

## **Beginning ABE – Geometry**

#### *New to this level and/or domain:*

There are no concepts added to the new Beginning ABE – Geometry.

#### Removed from this level and/or domain:

• Analyze characteristics and properties of 2- and 3-D shapes. (This is subsumed by "describe similarities and differences of 2- and 3-D shapes.)

## **Beginning ABE – Measurement**

*New to this level and/or domain:* 

• There are no concepts added to the new Beginning ABE – Measurement

## Removed from this level and/or domain:

- Apply knowledge of basic measurement concepts.
- Recognize, describe, and estimate attributes of length, height, weight, and temperature.

#### **Major Shifts in ABE I:**

When comparing the new ABE I standards to the existing we find the largest number of additional concepts in the ABE I – Number Sense, where we now find fluency, estimation, and properties addressed. Again, this reflects the indepth focus on numeracy in the early levels. In all other domains at this level there are very few additions and/or deletions although the probability concepts have been moved to higher levels to reflect the CCSS. Detailed descriptions of how the new standards are changed when compared to the existing can be found below:

#### ABE I - Number Sense

- Add up to four-digit numbers using strategies based on place value and properties.
- Fluently add and subtract within 1000.
- Select appropriate operational and relational symbols.
- Understand special properties of 0 and 1 in multiplication and division.
- Find missing element in a number sentence.
- Understand the concept of decimal numbers.
- Understand fractions as parts of wholes and as positions on a number line.
- Compare like-denominator fractions, using appropriate relational symbols.
- Describe differences between estimates and actual calculations.

- Estimate sums or differences of decimals to two places.
- Use commutative and associative properties for addition and multiplication. (Moved to Number from Algebra)

#### Removed from this level and/or domain:

- The limitation of "to 100,000" was reduced to "to 1000."
- Understand the meaning of the four operations and use their inverses to check results.
- Multiply and divide, by whole numbers, money amounts in decimal notation. (Moved to ABE II Number)
- Round whole numbers to the nearest multiple of 10.

## ABE I – Data Analysis

#### *New to this level and/or domain:*

- Creates and conducts surveys, interviews, and observations to collect and display data.
- Use line, circle, and bar graphs to summarize and display results of probability experiments.

## Removed from this level and/or domain:

- Predict the likelihood of outcomes in simple probability experiments.
- Compare the outcome of a probability experiment to the prediction.

## ABE I - Algebra

#### *New to this level and/or domain:*

- Recognize and extend linear patterns using rules.
- Identify number patterns in skip counting.

## Removed from this level and/or domain:

- Create a variety of patterns, formulate generalizations, and make predictions. (Finding general patterns was moved to ABE II Algebra. This level retained linear patterns only.)
- Select appropriate symbols and properties to represent and solve simple number relationships. (Moved to Number)
- Translate a problem into a number sentence and solve. (Moved to Number)
- Use the commutative an associative properties for addition and multiplication. (Moved to Number)

## **ABE I – Geometry**

*New to this level and/or domain:* 

- Partitions shapes into equal parts and express the areas as fractions of the whole.
- Identify shapes under transformations.

#### Removed from this level and/or domain:

• Measure angles by degrees and use correct related terminology.

#### ABE I - Measurement

New to this level and/or domain:

• There are no concepts added to the new ABE I – Measurement

Removed from this level and/or domain:

- Use formulas to compute perimeters and areas of squares and rectangles.
- Select appropriate US customary units and tools.
- Compare equivalent US customary units.

## Major Shifts in ABE II:

When comparing the new ABE II standards to the old, we see that there are very few additions made to this level. In every domain the number of new concepts is either one or none. There are several standards that have been moved to higher levels, e.g., ratio and proportion have been moved ABE III. in order to focus the content and better align to the Common Core State Standards. Detailed descriptions of how the new standards are changed when compared to the old can be found below:

#### ABE II – Number Sense

*New to this level and/or domain:* 

• There are no concepts added to the new ABE II – Number Sense

## Removed from this level and/or domain:

- Use math symbols:  $\langle , \cdot, \%, *, (), \{ \}, \angle .$
- Determine prime factors and common factors. (Find all factors of a number from 1 to 100 remains.)
- Use a calculator to verify results.
- Represent fractions, mixed numbers, decimals, and percents in equivalent forms.
- Interpret percent as part of 100.
- Represent probabilities as *proportions*. (This was changed to *fractions*.)

## ABE II – Data Analysis

#### *New to this level and/or domain:*

• There are no concepts added to ABE II – Data Analysis.

## Removed from this level and/or domain:

- Predict a future event.
- Understand the meaning of mean or average.
- Compute averages from pictographs and circle graphs.

# ABE II – Algebra

#### *New to this level and/or domain:*

• Analyze and determine patterns: symbolic, arithmetic, and geometric. (Moved here from ABE III – Number. Also deleted from ABE I – Algebra.)

## Removed from this level and/or domain:

- Use parentheses to indicate order of operations. (Moved to ABE II Number Sense)
- Represent simple functions.
- Set up single-variable proportions.

## ABE II - Geometry

New to this level and/or domain:

• There are no concepts added to the new ABE II – Geometry.

Removed from this level and/or domain:

• Compare, contrast, and analyze relationships between properties of geometric shapes.

#### ABE II - Measurement

*New to this level and/or domain:* 

Calculate simple unit conversions.

Removed from this level and/or domain:

• All concepts from the existing ABE II are included in the new.

## **Major Shifts in ABE III:**

When comparing the new ABE III standards to the old we see a larger number of changes than for any other level. Every domain in ABE III has multiple new/removed concepts, with Number Sense showing the biggest losses and Algebra the biggest gains to align with the shifts in the Common Core State Standards. Detailed descriptions of how the new standards are changed when compared to the existing can be found below:

#### ABE III - Number Sense

*New to this level and/or domain:* 

- Convert between fractions, repeating decimals, and percents.
- Identify a pattern when multiplying or dividing by powers of 10.
- Estimate to check results when operating with integers, exponents, and roots.
- Recognize the degree of precision needed for a problem.
- Calculate weighted averages.
- Construct circle graphs using ratios, proportions, and percents. (This was moved here from Algebra.)

Removed from this level and/or domain:

- Understand divisibility rules.
- Plot, define, solve, and simplify using expressions involving absolute value.
- Use correct notation for radicals, exponents, degrees, approximately equal, and absolute value.
- Use correct order of operations when solving word problems.
- Describe geometric progressions.
- Demonstrate the inverse relationship between negative and positive exponents.
- Find positive whole powers of positive and negative rational numbers.
- Analyze and determine patterns: symbolic, arithmetic, and geometric. (Moved to ABE II Algebra)
- Select appropriate operation to solve problems.
- Use scientific calculator to perform operations and check results.

## **ABE III – Data Analysis**

#### New to this level and/or domain:

- Understand how additional data can affect measures of center.
- Design and predict outcomes for a probability experiment.
- Express outcomes for a probability experiment verbally and numerically.

## Removed from this level and/or domain:

- Analyze graphs using more than one data source.
- Construct circle graphs using ratios, proportions, and percents. (This was moved to Number Sense.)

## ABE III – Algebra

- Identify dependent and independent variables.
- Understand the concept of functions.
- Compare different representations of functions.
- Understand slope.
- Evaluate simple expressions for specific values of the variable(s), including absolute value.
- Describe how a change in one variable affects another.

• The new standards specify linear and proportional relationships that must be set up and solved. (The existing version was more generic with, "Write and solve word problems...).

## Removed from this level and/or domain:

- Construct and solve linear equations requiring all four operations.
- Solve equations and inequalities involving absolute value.
- Justify each step in solving a problem or simplifying an expression.

## **ABE III – Geometry**

#### *New to this level and/or domain:*

- Identify in which quadrant a given point is located.
- Construct triangles from three angles or sides; determine the number of possible triangles.
- Recognize and draw 2-dimensional representatives of 3-dimensional figures (nets, perspectives, and cross-sections).
- Solve real-world area, volume, and surface area problems.
- Uses correct geometric notation, including for line segments, angles, and congruence.
- Use scale drawings to compute actual lengths and areas and reproduce using different scales.

## Removed from this level and/or domain:

- Construct geometric figures.
- Identify and use the properties of angles.

#### ABE III - Measurement

- Use scientific notation to express large-scale measurements. (Small-scale measurements remain a part of the new standards.) Removed from this level and/or domain:
  - All concepts from the existing ABE III are included in the new.

#### **Major Shifts in ASE I:**

When comparing the new ASE I standards to the existing we find that Number Sense and Algebra have the largest number of changes. Number Sense now includes computation with polynomials and negative exponents, along with several deletions. In Algebra we find more additions than deletions to further strengthen the demands. Detailed descriptions of how the new standards are changed when compared to the old can be found below:

#### ASE I - Number Sense

*New to this level and/or domain:* 

- Compute fluently with polynomials.
- Solve problems involving operations with polynomials, algebraic fractions, and mixed expressions.
- Demonstrate the inverse relationship between positive and negative exponents.

## Removed from this level and/or domain:

- Compute fluently and accurately with real numbers.
- Understand the uses and limitations of mathematical symbols.
- Demonstrate the inverse relationship between squaring and finding the square root.
- Multiply and divide numbers in scientific notation.
- Understand the inverse relationship between exponents and logarithms. (Moved to ASE II.)
- Use a scientific calculator as appropriate.

## **ASE I – Data Analysis**

- Identify sampling errors in data collection.
- Describe how the size of a sample affects the validity of predictions.
- Justify predictions based on data.
- Compare two data sets using measures of center and spread.
- Removed from this level and/or domain:
- Know why a specific measure of center provides the most useful information in a context.
- Recognize standard deviation.
- Determine probabilities of dependent and independent events.

## ASE I – Algebra

#### New to this level and/or domain:

- Create a table from a linear function.
- Graph a line given in slope-intercept, point-slope, and standard forms.
- Emphasize domain and range when evaluating or operating on a function.
- Justify each step in a solution of a word problem.
- Identify minimum, maximum, intercepts, zeros, and increasing/decreasing intervals for a quadratic equation.
- Interpret the parameters in an exponential function.
- Determine whether two lines are parallel, perpendicular, or coincidental.
- Solve equations and inequalities involving absolute value.
- Solve an equation for one variable in terms of the others.
- Solve everyday problems involving equations, constraints, or inequalities.

#### Removed from this level and/or domain:

- Create, describe, extend, and generalize a variety of patterns (linear, quadratic, exponential).
- Convert between function representations: symbols, tables, and graphs.
- Operate with rational expressions and functions. (Moved to ASE II.)

## **ASE I – Geometry**

#### *New to this level and/or domain:*

- Apply properties of lines to determine geometric characteristics and solve geometric problems.
- Identify elements of 3-dimensional objects and describe how two or more objects are related in space.
- Use properties of special triangles to solve problems (e.g. isosceles, equilateral, 30-60-90, 45-45-90)
- Apply formulas for volume of prisms, rectangular solids, and cylinders.

## Removed from this level and/or domain:

• Define and graph any ordered pair on coordinate plane.

#### ASE I – Measurement

New to this level and/or domain:

• There are no concepts added to the new ASE I.

Removed from this level and/or domain:

• All concepts from the existing ASE I included in the new.

#### **Major Shifts in ASE II:**

When comparing the new ASE II standards to the existing we find that the Measurement domain no longer exists as all the concepts students need have been learned. In addition what was formerly under Logic and Reasoning has now been moved to Geometry and to a new section of the standards called Mathematical Practices. The biggest changes at this level are in Data Analysis and Geometry, where each domain shows five deleted concepts (some of which are addressed in earlier levels). Detailed descriptions of how the new standards are changed when compared to the existing can be found below:

#### ASE II - Number Sense

*New to this level and/or domain:* 

- Understand the inverse relationship between exponents and logarithms. (Moved here from ASE I.)
- Solve problems involving logarithms and exponents.
- Apply number theory concepts for real and complex numbers.
- Perform operations with complex numbers.

Removed from this level and/or domain:

• All concepts from the existing ASE II are included in the new.

## **ASE II – Data Analysis**

*New to this level and/or domain:* 

There are no concepts added to the new ASE II.

#### Removed from this level and/or domain:

- Construct and draw inferences from data gathered and summarized (using measures of center, spread, and correlation) from real-life situations.
- Organize data into frequency chats, stem-and-leaf, box-and-whisker, scatter plots and matrices. Determine outliers.
- Evaluate the effect of sampling on data collections and statistical claims.
- Apply curve-fitting to make predictions from data.
- Explain the concept of a random variable.

## ASE II - Algebra

#### New to this level and/or domain:

- Understand the relationship between solutions, zeros, intercepts, and factors of a polynomial.
- Perform operations with rational functions. (Moved from ASE I)
- Solve linear systems using Cramer's rule and matrices.
- Recognize which type of expression best fits a context or application.

## Removed from this level and/or domain:

• All concepts from the old ASE II are included in the new.

## **ASE II – Geometry**

#### *New to this level and/or domain:*

- Make conjectures based on triangle Inequality and other inequalities and use them to solve problems.
- Write the purpose of an algorithm.

## Removed from this level and/or domain:

- Determine the effects of transformations on linear and area measurements and sketch the resultant figure.
- Apply transformations to practical situations.
- Determine the validity of arguments. (Formerly in ASE II Logic and Reasoning)
- Formulate counterexamples and use indirect proof. (Formerly in ASE II Logic and Reasoning)
- Determine if the converse of a statement is true. (Formerly in ASE II Logic and Reasoning)

#### ASE II – Measurement

• ASE II does not have a Measurement domain

## **Mathematical Practices**

The Standards for Mathematical Practice have been added to the new Arizona standards. They describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

## **Strand:** Mathematical Practices

The learner exhibits adaptive reasoning, strategic competence, and productive dispositions when solving problems with mathematics in Indicators A-E.

	Beginning ABE Sub-Indicator	Α	BE I Sub-Indicator	AE	BE II Sub-Indicator	AE	BE III Sub-Indicator	AS	SE I Sub-Indicator	AS	E II Sub-Indicator
<ol> <li>2.</li> <li>3.</li> </ol>	Makes sense of problems and persevere in solving them. Reasons abstractly and quantitatively. Constructs viable arguments and critique the reasoning	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> </ol>	Makes sense of problems and persevere in solving them. Reasons abstractly and quantitatively. Constructs viable arguments and critique the reasoning of others. Models with	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> </ol>	Makes sense of problems and persevere in solving them. Reasons abstractly and quantitatively. Constructs viable arguments and critique the reasoning of others. Models with	1. 2. 3.	Makes sense of problems and persevere in solving them. Reasons abstractly and quantitatively. Constructs viable arguments and critique the reasoning of others. Models with mathematics.	<ol> <li>2.</li> <li>3.</li> <li>4.</li> </ol>	Makes sense of problems and persevere in solving them. Reasons abstractly and quantitatively. Constructs viable arguments and critique the reasoning of others. Models with mathematics.	2.	Makes sense of problems and persevere in solving them. Reasons abstractly and quantitatively. Constructs viable arguments and critique the reasoning of others.
4.	of others.  Models with mathematics.	5.	mathematics. Uses appropriate tools strategically.	5.	mathematics. Uses appropriate tools strategically.	5.	Uses appropriate tools strategically.	5.	Uses appropriate tools strategically.	4.	Models with mathematics.
<ul><li>5.</li><li>6.</li></ul>	Uses appropriate tools strategically. Attends to precision.	6. 7.	Attends to precision.  Looks for and makes use of structure.	6. 7.	Attends to precision.  Looks for and makes use of structure.	6. 7.	Attends to precision. Looks for and makes use of structure.	6. 7.	Attends to precision. Looks for and makes use of structure.	<ul><li>5.</li><li>6.</li></ul>	Uses appropriate tools strategically. Attends to precision.
7. 8.	Looks for and makes use of structure. Looks for and expresses regularity in repeated reasoning.	8.	Looks for and expresses regularity in repeated reasoning.	8.	Looks for and expresses regularity in repeated reasoning.	8.	Looks for and expresses regularity in repeated reasoning.	8.	Looks for and expresses regularity in repeated reasoning.	7. 8.	Looks for and makes use of structure. Looks for and expresses regularity in repeated reasoning.

## Strand: Number Sense

Indicator A: The learner develops and applies number sense to solve a variety of real-life problems and to determine if the results are reasonable.

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
Number Sense:	Number Sense:	Number Sense:	Number Sense:	Properties of Real	Properties of Natural,
Recognizes concrete	1. Counts, reads, and	1. Represents, orders, and	1. Finds, plots and orders	Numbers:	Rational, Real, and

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
and symbolic representation of whole numbers. 3.OA.8, 4.OA.3 <sup>1</sup> 2. Counts to 100 by 1s. K.CC.1  3. Counts within 1000 and skip counts by 2's, 5's, and 10's to at least 100. 3.NBT.2  4. Counts forward beginning at a given number in a sequence. 1.NBT.1  5. Compares and orders numbers to 100, using the number line. 2.MD.6  6. Understands the meaning of addition and subtraction and uses the inverse relationship between addition and subtraction to solve problems and check results. 1.OA.4, 1.OA.5, 1.OA.7, 1.OA.8	writes whole numbers to 1000. 4.NBT.2  2. Identifies place value and represents numbers to 1000 using various forms, including expanded notation. 4.NBT.2  3. Compares and orders whole numbers to 1000 on the number line. 2.NBT.4, 2.MD.6  4. Recognizes and uses sets to which a number may belong (odds, evens, multiples and factors). 4.OA.4  5. Selects appropriate operational and relational symbols to make an expression true. 5.OA.1, 6.EE.2a, 6.EE.4	compares very large and very small positive numbers (decimals, fractions, mixed numbers) using a number line. 8.EE.3, 4.NBT.2, 5.NBT.3  2. Understands the place value of whole numbers and decimals. 5.NBT.1	integers, fractions, mixed numbers, and decimals (both positive and negative) on the number line. 5. NBT. 3, 5. NBT. 3b, 6. NS. 6, 6. NS. 6ca, 6. NS. 6c, 6. NS. 7  2. Expresses a quantity as an equivalent fraction, decimal, and percent. 4. NF. 1, 4. NF. 5, 4. NF. 6, 5. NF. 3, 6. RP. 3c,  3. Understands number theory concepts up to rational numbers and their subgroups.  4. Expresses rational numbers as fractions, finite or repeating decimals, and percents; converts between these. 4. NF. 1, 4. NF. 5, 4. NF. 6, 5. NF. 3, 6. RP. 3c,  5. Demonstrates an understanding of absolute value. 6. NS. 7c,  6. Uses number concepts including primes, factors, and multiples to solve problems with fractions. 4. OA. 4,	1. Describes the properties of and compares the relationship between real, rational, irrational, integer, and whole numbers. 8.NS.1, 2. Uses the properties of operations on real numbers (commutative, associative, distributive, identity, inverse and closure properties). 7.NS.2c, 7.NS.1d, 7.NS.2d,	Complex Numbers:  1. Knows and uses the properties of operations on real numbers, including nth power and nth root. N.RN.2  2. Understands and applies number theory concepts including real and complex numbers. N.CN.2

----

<sup>&</sup>lt;sup>1</sup> The citations that appear after each Arizona standard represent the Common Core State Standard(s) to which they relate. Where there is no citation, no corresponding Common Core State Standard exists.

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
Computation:  7. Knows and uses addition and subtraction facts to 12 with fluency. 1.OA.6  8. Selects the appropriate operation (+, -, =) to solve specific problems involving-whole numbers. 1.OA.1, 1.OA.6  9. Uses and understands the value of US coins and currency, including recognizing currency symbols and identifying the value of combinations of pennies, nickels, dimes, quarters, half-dollars, and dollar bills. 2.MD.8  10. Uses combinations of different coins and currency to make change. 2.MD.8, 4.MD.2  11. Solves one- and two-digit problems using addition and subtraction, written in both vertical and horizontal notation, with and without regrouping. 1.NBT.4  12. Writes and solves number sentences from problem situations that express relationships,	Computation:  6. Adds up to four two-digit numbers using strategies based on place value and properties of operations. 4.NBT.4  7. Solves problems involving addition and subtraction of multidigit whole numbers between 0 and 100,000. 4.OA.3  8. Fluently adds and subtracts within 1000, using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 4.NBT.4  9. Masters the multiplication and division facts through 12. 3.OA.7  10. Solves problems involving multiplication and division of two-digit whole numbers without remainders. 3.OA.3  11. Selects appropriate operation (i.e. addition, subtraction, multiplication, and division) to solve one-step word problems and	Computation:  3. Multiplies and divides whole numbers up to 1,000, with remainders represented as fractions.  4.NBT.5, 4.NBT.6  4. Find all factors for a whole number in the range 1–100. 4.OA.4  5. Identifies and uses correct order of operations in problems that include a variety of mathematical symbols.  6.EE.2c  6. Multiplies and divides money amounts in decimal notation using whole-number multipliers and divisors.  4.MD.2  7. Selects the appropriate operation to solve problems and determines whether the results are reasonable. 3.OA.8,  4.OA.3	Computation:  7. Understands and applies rules for the addition, subtraction, multiplication and division of integers (signed numbers).  6.NS.5, 7.EE.3  8. Uses and models computations with positive and negative integers, decimals, fractions, mixed numbers, and percents.  7.NS.1d, 7.NS.2c  9. Solves word problems using positive and negative rational numbers. 7.NS.3  10. Calculates given percentages of quantities involving discounts, sales tax, interest-earning savings plans, tips, down payments, and so on.  6.RP.3c, 7.RP.3  11. Calculates the percentage increase or decrease of a quantity. 7.RP.3  12. Understands the concept of cross-multiplication.  7.RP.3  13. Calculates problems with a combination of fractions, decimals, percentages, and exponents using a scientific calculator to	Computation and Reasoning:  3. Solves problems involving ratio units such as miles per hour, dollars per pound, or persons per square mile. 6.RP.3b  4. Applies computation, estimation, and/or proportions to solve multi- step word problems involving integers, rational numbers, exponents, square roots, and scientific notation. 6.RP.3, 7.NS.3, 8.EE.2, 8.EE.4  5. Demonstrates fluency in computations with polynomials. A.APR.1  6. Uses addition, subtraction, multiplication and division to solve problems involving monomials, binomials, polynomials, and algebraic fractions and mixed expressions. A.APR.1, F.BF.1b	Computation and Reasoning 3. Simplifies numerical expressions with powers and roots, including fractional and negative exponents. 8.EE.2 4. Uses the definition of logarithms to translate between logarithms in different bases. F.LE.4 5. Understands and uses the inverse relationship between exponents and logarithms. F.BF.5 (+) 6. Understands and applies the laws of logarithms. F.LE.4 7. Solves problems involving logarithms and exponents. F.BF.5 (+) 8. Performs arithmetic operations with complex numbers. N.CN.5 (+)

Beginning ABE	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
Beginning ABE Sub-Indicator  including +, -, <, >, =   with numbers between   0 and 20. 1.NBT.3  13. Uses the concepts of   commutative and   associative rules for   addition to simplify   mental calculations and   to check results.   1.OA.3	determines the reasonableness of results. 3.OA.8  12. Understands the special properties of 0 and 1 in multiplication and division. [See Table 3 in Glossary/Appendix]  13. Understands the concept of decimal numbers. 4.NF.5, 4.NF.6  14. Estimates and computes the sum or difference of whole numbers and decimals to two places. 4.MD.2  15. Finds the missing element in a number sentence. 1.OA.1, 3.OA.4  16. Writes and solves number sentences from word problems that use addition, subtraction, multiplication and division. 3.OA.8	ABE II Sub-Indicator	check results. 7.NS.3, 6.EE.1, 6.RP.3c, 8.EE.1, MP.5	ASE I Sub-Indicator	ASE II Sub-Indicator
	division. 3.OA.8  17. Uses the concept of commutative and associative properties of addition and				
	multiplication to simplify calculations and check results. 3.OA.5				
	Rounding and Estimation: 18. Makes reasonable	Rounding and Estimation: 8. Estimates and rounds	Estimation and Reasonableness of Results:	Estimation and Reasonableness of Results:	

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
	estimates3.OA.8, 4.OA.3  19. Describes differences between estimates and actual calculations.  20. Rounds whole numbers through 1000 to the nearest multiple of 100 or 10. 3.NBT.1  21. Uses estimation to check the reasonableness of results of whole number computation. 4.OA.3	very large (e.g., millions) and very small (e.g., thousandths) numbers. 8.EE.3  9. Uses estimation to check results involving fractions, mixed numbers and decimals and rounds numbers where appropriate.  5.NF.2	<ul> <li>14. Recognizes the degree of precision needed in a problem. [See MP.6]</li> <li>15. Uses estimation to check the results of problems involving integers, exponents, and roots.</li> <li>7.EE.3, 8.NS.2, 8.EE.1</li> </ul>	7. Gives answers to numerical problems to a specified degree of accuracy. [See MP.6]	
	Fractions:  22. Understands fractions as parts of unit wholes, as parts of a set, and as locations on the number line. 3.NF.1, 3.NF.2  23. Matches a fraction (written as a number or in words) to a pictorial representation of halves, thirds, fourths and tenths. 2.G.3  24. Represents and describes equivalencies of common fractions (halves, thirds, fourths, tenths). 3.NF.3b  25. Compares two fractions with the same denominator using the symbols, >, =, or <.	Fractions: 10. Simplifies fractions to lowest terms. 3.NF.3b, 4.NF.1 11. Adds, subtracts, multiplies, and divides fractions and decimals, including mixed numbers. 4.NF.3c, 5.NBT.7	Ratios and Proportion:  16. Uses ratios and proportions in the solution of problems involving unit rates and scale drawings, markups, commissions, profits, simple interest using appropriate notations: (a/b, a to b, a:b). 6.RP.3, 7.RP.3  17. Calculates weighted averages such as course grades, consumer price indexes, and sports ratings. S.MD.2(+)  18. Constructs circle graphs using ratios, proportions and percents. 3.MD.3		

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
	4.NF.2  26. Adds and subtracts simple fractions with like and unlike denominators (e.g., 1/8 + 3/8 = 1/2).  4.NF.3d, 5.NF.1				
			Exponents and Square Roots:  19. Identifies patterns when multiplying or dividing a decimal by a power of 10. 5.NBT.2  20. Understands and uses exponents, roots, and scientific notation.  5.NBT.2, 8.EE.3, 8.EE.4  21. Interprets and evaluates expressions involving integer exponents and perfect roots and approximation of imperfect roots. 8.EE.1, 8.EE.2  22. Names the square root of a number with a perfect square. 8.EE.2  22. Estimates the square root of any whole number to the nearest whole number. 8.NS.2  23. Multiplies and divides expressions involving exponents with a common base. A.SSE.3c, A, APR.7(+)  24. Evaluates rational	Exponents and Square Roots:  8. Demonstrates the inverse relationship between positive and negative exponents (reciprocals).  8. EE.1	

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
			numbers raised to the		
			positive whole number powers. 6.EE.1		

Strand: Data Collection and Probability

Indicator B: The learner applies data collection, data analysis, and probability to interpret, predict and/or solve real-life problems

Sub-indicator		ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
Analysis: Anal	nta Collection and nalysis:  Interprets, and constructs tables, charts, and graphs (pictographs, horizontal and vertical bar graphs, line graphs, circle graphs) with appropriate scales using data with up to three categories. 1.MD.4, 2.MD.10, 3.MD.3  Creates and conducts surveys, interviews, and observations and uses results to construct simple charts and graphs with appropriate scales. 7.SP.2	Data Collection and Analysis:  1. Collects, records, organizes, and displays data using graphs with appropriate scales (e.g., pictographs, histograms, and bar graphs).  3.MD.3, 4.MD.4, 6.SP.4,  2. Interprets and analyzes data presented in a graph which includes symbols in a key representing multiple units.	Data Collection and Analysis:  1. Organizes data and constructs appropriate graphic form to best depict the data. 4.MD,4, 5.MD.2, 6.SP.4  2. Interprets and analyzes graphs. 4.MD.4, 5.MD.2, 6.SP.5d  3. Formulates questions from graphs, tables, and charts. 6.SP.1, 6.SP.2  4. Uses results of data to predict future events. 7.SP.2, 7.SP.6	Data Collection and Analysis:  1. Identifies data that represent sampling errors and explains why the sample (and the display) might be biased. 7.SP.1, S.IC.3  2. Describes how the relative size of a sample affects the validity of the predictions. 7.SP.6  3. Describes the characteristics and limitations of the samples; differentiates between a sampling and a census. 7.SP.1, S.IC.3  4. Organizes collections of data into frequency charts, histograms, dot plots, stem-and-leaf plots,	Data Collection and Analysis:  1. Uses the forms of stem-and- leaf plots or box-and-whisker plots to compare two sets of data. S.ID.2  2. Represents two numerical variables on a scatterplot and informally describes how the data points are distributed and any apparent relationship that exists between the two variables. S.ID.6b  3. Draws a line or a curve which closely fits a scatter plot. S.ID.6a  4. Determines, from a given plot of data, whether it has strong or weak, positive or

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
				whisker plots, and matrices and determines outliers. 6.SP.4, 8.SP.4  5. Makes valid inferences and evaluates the reasonableness of conclusions drawn from data. 7.SP.1, 7.SP.2, S.IC.3  6. Given a scatter plot draw the line of best fit. 8.SP.2, S.ID.6a, S.ID.6b, S.ID.6c  7. Formulates and justifies predictions from a given set of data. S.IC.4, S.IC.5  8. Determines possible graphical distortions and manipulation of statistics. S.ID.9, S.IC.6	S.ID.8  5. Defines and recognizes standard deviation. S.ID.2  6. Draws conclusions about the "spread" of data given the variance and standard deviation (e.g., compare sets of data with the same central tendency, but with different variance). S.ID.2, S.ID.3  7. Designs a statistical experiment based on a given hypothesis. 7.SP.6, 7.SP.8c, S.CP.4  8. Identifies and analyzes graphical distortions, possible representations of bias, and manipulation of statistics. S.IC.6  9. Differentiates between a biased and an unbiased sample; recognizes the impact of interpreting data from a biased sample. S.IC.3
		Measures of Central Tendency: 3. Computes and interprets	Measures of Central Tendency: 5. Calculates mean,	Measures of Central Tendency: 9. Understands the	Properties of the Normal Curve: 10. Describes the central
		the average based on	median, mode and	meaning of, and	tendency
		information from bar	range. 6.SP.5c	computes the	characteristics of the
		graphs. 6.SP.5d	6. Compares data sets using	minimum, the lower	normal curve. S.ID.4

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
			the concepts of mean, median, mode and range. 6.SP.5c, 7.SP.4  7. Understands why a specific measure of central tendency provides the most useful information in a given context.  8. Understands how additional data added to data sets may affect the computations of measures of central tendency.	quartile, the median, the upper quartile, and the maximum of a data set. 6.SP.5c, S.ID.2  10. Compares two data sets using statistics appropriate to the shape of the data related to center (median, mean) and spread (interquartile range). 7.SP.3, 7.SP.4, S.ID.2	<ul> <li>11. Describes, in general terms, the normal curve and uses its properties to answer questions about sets of data that are assumed to be normally distributed. S.ID.4</li> <li>12. Determines if data gathered from a realworld situation fits a normal curve. S.ID.4</li> <li>13. Determines the variance and the standard deviation of a normally distributed random variable. S.ID.4</li> </ul>
	Probability: 3. Summarizes and displays the results of probability experiments in a clear and organized way (e.g., line graphs, circle graphs, bar graphs). 7.SP.8b	Probability:  4. Organizes (e.g., sorts, sequences, tallies) data from a simple one- step probability experiment.  7.SP.7b  5. Represents probabilities as ratios, decimals, and fractions between 0 and 1, and percentages between 0 and 100; verifies that the probabilities computed are reasonable. 7.SP.5  6. Uses percentages and fractions to describe the probability of an event.  7.SP.5	Probability:  9. Expresses outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4; 3/4). 7.SP.5  10. Understands the difference between theoretical and experimental probability. S.IC.2,  11. Designs and predicts outcomes of a probability experiment based on theoretical probability. S.IC.2  12. Understands the difference between independent and dependent events.	Probability:  11. Applies the set operations of union and intersection and the concept of complement, universal set, and disjoint sets and uses them to solve problems, including those involving Venn diagrams. S.CP.1  12. Uses basic combinatorics ("fundamental counting principle"). 7.SP.8	Probability:  14. Knows the definition of the notion of independent events and uses the rules for addition, multiplication, and complementation to solve for probabilities of particular events.  S.CP.2  15. Knows the definition of conditional probability and uses it to solve for probabilities in finite sample spaces.  S.CP.3, S.CP.4,  S.CP.5, S.CP.6  16. Uses combinatorics

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
			S.CP.2  13. Understands that the probability of two disjoint events occurring is the sum of the two individual probabilities.  14. Understands that the probability of one event following another, in independent trials, is the product of the two probabilities.  15. Finds probabilities of compound events using organized lists, tables, tree diagrams, and simulations. 7.SP.8b, 7.SP.8c		(e.g., fundamental counting principle, permutations, and combinations) to solve problems, including computing geometric probabilities and probabilities of compound events. S.CP.9  17. Determines probabilities through experiments and simulations and compares the results with predictions. S.IC.2

Strand: Algebra

Indicator C: The learner applies algebraic concepts and methods to explore, analyze or solve real-life problems.

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
	Patterns:	Solving Simple Equations	Representing Linear	Properties of Functions:	<u>Properties of Functions</u> :
	1. Identifies the pattern in	1. Writes and solves simple	<u>Functions:</u>	1. Understands the	1. Identifies the
	skip counting (e.g., 2,	equations with one	1. Identifies dependent and	concepts of function:	dependent and
	4, 6 – add 2 to each	variable. 4.OA.3	independent variables.	its domain and range.	independent variables
	number). 3.OA.9	2. Interprets and evaluates	6.EE.9	8.F.1, F.IF.1	in a function derived
	2. Creates, analyzes and	mathematical	2. Sets up proportional	2. Evaluates a function at	from a real- life
	explains repeating and	expressions, using order	relationships using	a specified point in its	situation. 6.EE.9
	growing patterns.	of operations. 6.EE.2c	dependent and	domain, and performs	2. Expresses
	4.OA.5	3. Knows and understands	independent variables.	operations on	generalizations and
	3. Explains the logic of a	that equals added to	7.RP.2	functions with	models, analyzes, and
	pattern and formulates	equals are equal and that	3. Understands the concept	emphasis on the	interprets real- world

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
	generalizations to make predictions about what element comes next. 5.OA.3  4. Extends and recognizes a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4's or by multiplying the number of horses by 4). 8.SP.1	equals multiplied by equals are equal.  4. Identifies the variable and the constant in word problems. 6.EE.6, 7.EE.4  5. Analyzes and determines the patterns for extending symbolic, arithmetic, and geometric progressions, e.g., ABBCCC; 1, 5, 9, 13; 3, 9, 27, Pascal's Triangle.  4.OA.5, 5.OA.3, 8.SP.1	of a function. 8.F.1  4. Identifies and compares properties of functions in different representations. Ex: table of values, algebraically, graphically, verbal description and comparing rate of change between the two functions 8.F.2  5. Solves problems involving linear relationships with integer values and graphs the resulting ordered pairs of integers on a grid. 8.F.4, 8.F.5  6. Understands the concept of slope. 8.SP.3	domain and range. F.IF.2  3. Represents functions in different ways (e.g., words, formulas, tables and graphs). 8.F.2, A.CED.2, F.BF.9  4. Interprets the parameters in an exponential function-F.LE.5	phenomena using polynomial, rational, logarithmic, or exponential functions. A.CED.1, A.REI.11, S.ID.6a  3. Determines the effects of transformations on functions. F.BF.3  4. Solves everyday problems that can be modeled using various functions, absolute values, and square roots. Including compound interest, exponential growth and decay, and direct and inverse variation problems. A.CED.1, A.REI.11  5. Understands and identifies the properties of rational functions: its domain, its range, and asymptotes. F.IF.7d  6. Performs basic arithmetic operations with rational functions. A.APR.6, A.APR.7(+)  7. Understands the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
					polynomial. A.APR.3, A.REI.11, F.IF.7c
			Solving Linear Equations and Inequalities:  7. Uses variables in simple expressions, computes the value of the expression for specific values of the variable. 6EE.2c  8. Translates word problems into expressions using correct mathematical symbols (e.g., <, >, =).  7.EE.4  9. Represents and describes how changing the value of one variable in a relationship results in a change in another. 8.F.1  10. Identifies and uses correct order of operations for algebraic expressions. 8.EE.2c  11. Uses the associative, commutative, identity, inverse, and distributive properties to simplify calculations. 7.NS.1d, 7.NS.2c  12. Simplifies an expression by combining like terms (e.g., 3x + 2 + 2x + 3 simplifies to 5x + 5).  8.EE.7b  13. Solves word problems involving a linear	Solving Linear Equations and Inequalities:  5. Creates a table from a linear function and graphs the function and identifies the x- and y-intercepts. 8.F.4  6. Finds the slope using the point-slope formula.  7. Graphs linear equations given in slope-intercept, point-slope, and standard form. F.IF.7a  8. Determines from two linear equations whether the lines are parallel, perpendicular or coincide.  9. Derives linear equations by using the point-slope or slope-intercept formulas.  10. Solves word problems using equations or inequalities and justifies each step in the solution. 6.EE.7, 6.EE.8, A.REI.1  11. Solves equations and inequalities to include absolute values. A.REI.11  12. Solves variables in terms of other	Solving Equations and Inequalities, 8. Solves equations involving radical functions. A.REI.2 9. Simplifies expressions with powers and roots, including fractional exponents. N.RN.2

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
			equation or inequality. 6.EE.7, 6.EE.8 14. Evaluates absolute value expressions. 6.NS.7c	variables. A.CED.4  13. Solves linear equations with fractional coefficients (e.g., Celsius to Fahrenheit conversion). 8.EE.7b	
				Solving Systems of Equations:  14. Solves systems of equations and inequalities, using substitution, elimination, and graphing. 8.EE.8b  15. Solves everyday problems that can be modeled using systems of linear equations or inequalities. 8.EE.8c  16. Solves everyday problems involving equations, constraints, or inequalities. 6.EE.8, 8.EE.8c, A.CED.3	Solving Systems of Equations: 10. Performs operations on matrices and uses matrices in applications. N.VM.7, N.VM.8 11. Solves systems of linear equations by using Cramer's rule and matrices A.REI.9(+)
				Solving Quadratic Equations:  17. Applies basic factoring techniques (e.g., difference of 2 squares, common factor for all terms, perfect squares of binomials). A.SSE.3a  18. Plots quadratic functions on an x- and y- coordinate system.  F.IF.7a	Solving Quadratic Equations:  12. Finds solutions to quadratic equations with real coefficients and real or complex roots graphically, by factoring, completing the square, or using the quadratic formula. N.CN.7

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
				<ul> <li>19. Solves quadratic equations by inspection (e.g., for x² = 49), taking square roots, factoring, or using the quadratic formula as appropriate. F.IF.8a</li> <li>20. Graphs quadratic equations (e.g., for x² = 49) and identifies relative maximum and minimum values, intercepts, zeros, and intervals over which it is increasing or decreasing. F.IF.7a</li> <li>Calculating with Polynomial Expressions:</li> <li>21. Adds, subtracts, multiples, and divides polynomials. A.APR.1</li> </ul>	Interpreting Equations and Inequalities Geometrically  13. Recognizes which type of expression best fits the context of a basic application (e.g., linear equation to solve distance/time problems; quadratic equation to explain the motion of a falling object; or compound interest as an exponential function).  F.BF.1, F.BF.1a

# Strand: Geometry

Indicator D: The learner uses geometric properties, relationships, and methods to identify, analyze and solve real-life problems

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
	7. Identifies shapes under rotation (turns), reflections (flips), translation (slides), and enlargement.  Describes direction of translations (e.g., left, right, up, down). 8.G.3	Angles: 6. Knows the definitions of right angles, acute angles, obtuse angles; identifies right angles in geometric figures; and determines whether other angles are greater or less than a right angle. 4.G.1, 4.G.2 7. Measures angles in whole-number degrees using a protractor and uses correct terminology	Angles: 4. Draws, and accurately measures angles between 0 and 360 degrees. 4.MD.6, 7.G.2 5. Classifies triangles by their angles and sides as equilateral, isosceles, scalene, or right. 4.G.2 6. Uses the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve	ASE I Sub-Indicator	ASE II Sub-Indicator
		to describe them. Sketches angles of a given measure. 4.MD.6	problems involving an unknown angle. 7.G.5  7. Knows that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° and uses this information to solve problems. 8.G.5  8. Knows and understands the Pythagorean theorem and its converse; uses it to find the length of the missing		
			side of a right triangle and the lengths of other		

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
			line segments. 8.G.6,		
			8.G.7		
Sub-indicator		Coordinate Geometry:  8. Uses first quadrant coordinate points to graph lines and simple figures. 5.G.2	line segments. 8.G.6, 8.G.7  Coordinate Geometry:  9. Graphs and identifies coordinate points in all four quadrants; identifies in which quadrant a given point is located. 6.NS.6b, 6.NS.8	Coordinate Geometry:  1. Applies properties of lines (slope and midpoint of a line segment and distance formula) to determine geometric characteristics and solve geometric problems. 6.G.3, 8.EE.6, 8.G.8  2. Draws and interprets transformations on figures in the coordinate plane such as translations, reflections, rotations, scale factors, and the results of successive transformations. 8.G.3, 8.G.4  3. Solves problems requiring transformations. 8.G.1, 8.G.2, G.CO.3, G.CO.5  4. Identifies elements of three- dimensional geometric objects	
				(e.g., diagonals of	
				rectangular solids) and describes how two or	
				more objects are	
				related in space (e.g.,	
				skew lines, the possible	

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
	ABL I Sub-indicator	ADE II Sub-III dicator		ways three planes might intersect).  5. Finds a linear equation describing a line from a graph or a geometric description of the line (e.g., by using the point-slope, slope y-intercept formulas, and average rate of change). 6.EE.9, 7.EE.4, 7.EE.4a  6. Demonstrates the ability to visualize solid objects and recognize their projections, cross sections, and graph points in 3-D. G.GMD.4,	AGE II GUD-IIIGICATOI
			Properties of Similarity and Congruence:  10. Knows correct geometric notation, including the notation for line segments, angles and congruence.  11. Describes the conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures. 8.G.2, (G.CO.6, G.CO.7, G.CO.8)  12. Determines if two figures are similar,		

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
			congruent or neither. 8.G.4, (G.SRT.2, G.SRT.5) 13. Solves problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scaled drawing at a different scale. 7.G.1		
			Concept of π and circular figures:  14. Knows common estimates of π (3.14, 22/7) and uses these values to estimate and calculate the circumference and the area of circles; compares with actual measurements. 7.G.4, 8.NS.2  15. Uses formulas to calculate the area of basic two-dimensional figures and volume of three-dimensional figures, including triangles, quadrilaterals, polygons, cubes, cylinders and right prisms. 7.G.4, 7.G.6  16. Solves real world mathematical problems involving area, volume, and surface area of two		

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
			and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, cylinders and right prisms. 6.G.2, 6.G.4, 7.G.6, 8.G.9,		
				Using Properties and Theorems to Make Conjectures and Solve Problems: 7. Identifies basic elements of geometric figures (e.g., altitudes, mid- points, diagonals, angle bisectors, perpendicular bisectors, central angles, radii, diameters, arcs, tangents, secants and chords of circles). 5.G.4 8. Applies properties of sides, diagonals, and angles in special polygons (e.g., isosceles triangles, parallelograms, and rhombuses); identifies their parts and special segments (e.g., altitudes, midsegments); determines the measure of interior angles for regular polygons. 8.G.5,	Using Properties and Theorems to Make Conjectures and Solve Problems  3. Understands the ideas behind simple geometric proofs and develops and writes simple geometric proofs (e.g., the Pythagorean theorem). 8.G.6, G.SRT.4  4. Applies the triangle inequality and other inequalities associated with triangles (e.g., the longest side is opposite the greatest angle) to make additional conjectures and to solve problems. 8.G.5, G.SRT.5  5. Uses properties and theorems about congruent and similar figures and about perpendicular and parallel lines to make additional conjectures

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	G.CO.9, G.CO.10, G.CO.11  9. Solves simple triangle problems using the triangle angle sum property and/or the Pythagorean theorem. G.SRT.8, G.CO.10  10. Explains a proof of the Pythagorean Theorem and its converse. 8.G.6  11. Applies congruence and similarity correspondences (e.g., ABC ≅ or ~ XYZ) and properties of the figures to find missing	and to solve problems. G.SRT.4, G.SRT.5, G.CO.9, G.CO.10, G.CO.11  6. Draws a simple valid conclusion from a given if then statement and/or a minor premise and places the statements in logical order.  7. Determines the validity of arguments, to include using Venn diagrams.M.P.3  8. Recognizes the difference between a
				parts of geometric figures. G.CO.8, G.SRT.5  12. Uses the properties of special triangles (e.g., isosceles, equilateral, 30°-60°-90°, 45°-45°-90°) to solve problems. 8.G.7,  13. Understands and applies the formulas for the volume of triangular prisms, rectangular solids and cylinders. 6.G.2, 7.G.6, 8.G.9	statement verified by mathematical proof (i.e., a theorem) and one verified by data (e.g., observation and experimentation). M.P.3  9. Lists related ifthen statements in logical order.  10. Constructs a simple informal deductive proof (e.g., write a proof of the statement: "You can fly from Bombay to Mexico City, given an airline schedule.")  8.G.6, G.CO.9, 10, and 11, G.SRT.4, 5,

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
					10 11. Produces a valid conjecture using inductive reasoning by generalizing from a pattern of observations.  A.APR.5, M.P.3 (see footnote) 12. Analyzes assertions about everyday life by using principles of logic (e.g., examine the fallacies of advertising). M.P.3 13. Determines the purpose of, an algorithm that explains a particular mathematical process (e.g., tell a younger child how to find the average of two numbers). 14. Writes the purpose of, an algorithm that explains a particular mathematical process.
					Right Triangles and Trigonometric Functions:  15. Knows the definitions of trigonometric functions and uses them to find the sine, cosine and tangent of the acute angles of a right triangle.

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
					G.SRT.6,  16. Solves simple right- triangle problems involving sine, cosine and tangent and uses an appropriate right- triangle trigonometric model to solve a real- life problem. G.SRT.8

Strand: Measurement

Indicator E: The learner applies knowledge of standard measurements to real-life situations.

Beginning ABE Sub-Indicator	ABE I Sub-Indicator	ABE II Sub-Indicator	ABE III Sub-Indicator	ASE I Sub-Indicator	ASE II Sub-Indicator
1. Identifies the unit of measure of each specific tool (e.g., ruler = inches; scale = pounds, ounces; watch = minutes, seconds, hours; thermometer = degrees). K.MD.1, K.MD.2, MP.2, MP.5  2. Selects and uses the appropriate tool (e.g., ruler, thermometer, measuring cup, scale, stop watch) to measure the given attribute (length, temperature, volume, weight, time) of an object or event using customary and	1. Expresses the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understands that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limited to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. 1.MD.2	1. Estimates a measurement (using US customary or metric units) of a given object or event and compares the estimation to actual measurement and justifies the reasonableness of the answer. 2.MD.3, 3.MD.2  2. Identifies the appropriate type of measurement for each attribute of an object or event and justifies answer (length - inches, feet and yards,	<ol> <li>Differentiates between perimeter, area, and volume of polygons and solids (using concrete and illustrative models); between weight and mass; between capacity and volume. 3.MD.2, 4.MD.3, 5.MD.4,</li> <li>Selects the degree of precision and accuracy required for a given situation. MP.6, N.Q.3</li> <li>Converts between Fahrenheit and Celsius.</li> <li>Uses formulas to calculate:         <ul> <li>Perimeter,</li> </ul> </li> </ol>	1. Calculates lateral areas, surface areas, and volumes of prisms, pyramids, spheres, cylinders, and cones, and relates these measures to each other using formulas (e.g., find the volume of a sphere with a specified surface area). 7.G.6, G.GMD.3  2. Solves real world problems using unit conversion and verifies the reasonableness of	

- metric measures. 2.MD.1, MP.5
- 3. Compares the length, weight, and height of two or more objects using direct comparison (e.g., which object is shorter, longer, lighter, heavier). K.MD.1, K.MD.2, 2.MD.4
- Tells and writes time from both analog and digital clocks to the nearest minute and/or five minutes, using a.m. and p.m. 2.MD.7
- Identifies the appropriate type of measurement for each attribute of an object or event:
  - Length inches, feet and yards
  - Capacity cups, gallons
  - Weight ounces, pounds, tons
  - Area square unit
  - Volume cubic unit
  - Time second, minute, hour, day, month, year, decade, century
  - Temperature degrees on Fahrenheit scale, degrees on Celsius scale K.MD.1, K.MD.2, MP.2, MP.5
- 3. Estimates the area and volume of solid figures using manipulatives (e.g., by covering them with squares or by counting the number of cubes that would fill them). 3.MD.5b, 5.MD.4, 6.G.2
- 4. Understands and uses formulas to compute perimeter and area of square and rectangular figures. 3.MD.7,

- millimeters, centimeters, meters, kilometers)
- Capacity cups, gallons, milliliters, liters
- Weight ounces, pounds, tons, grams, kilograms
- Area square unit K.MD.1, K.MD.2, MP.2, MP.5
- Volume cubic unit
- 3. Calculates simple unit conversions. 4.MD.2
- 4. Knows relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. 4.MD.1
- 5. Compares units of measurement to determine more/less relationships using US customary and metric units (e.g. quart < liter).

- circumference, and area of common geometric figures such as parallelograms, trapezoids, circles, and triangles 3.MD.8, 4.MD.3, 7.G.4
- Area of irregular shapes made up of triangles and quadrilaterals 3.MD.7d, 6.G.1
- Surface area of rectangular containers 7.G.6
- Volume of rectangular containers, cylinders and triangular prisms.
   7.G.6
- 5. Uses scientific notation to express units of measurement in large scales, e.g., distance of sun from earth = 93,678,912 miles = 9.3678912 x 10<sup>7</sup> 8.EE.3, 8.EE.4
- 6. Uses scientific notation to express units of measurement in small scales using negative exponents. 8.EE.3, 8.EE.4
- 7. Uses similarity and proportionality to

- the answer. 6.RP.3, N.Q.1
- 3. Understands how a change in one attribute affects the change in another attribute (e.g., how changing the radius or height of a cylinder affects its surface area or volume). 7.G.1, 7.G.6
- 4. Understands that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

3.MD.8 5. Estimates a	solve measurement problems. G.SRT.2	
measurement of a given object or event and compares the estimation to actual measurement. 2.MD.3,	problems. G.SR1.2	
3.MD.2		